

Convert the polar equation $r = 3 + \cos 2\theta$ to rectangular.

SCORE: ____ / 15 PTS

$$r = 3 + \cos^2 \theta - \sin^2 \theta$$

$$r = 3 + \frac{x^2}{r^2} - \frac{y^2}{r^2}$$

$$r^3 = 3r^2 + x^2 - y^2$$

$$(x^2 + y^2)^{\frac{3}{2}} = 3x^2 + 3y^2 + x^2 - y^2$$

$$= 4x^2 + 2y^2$$

$$(x^2 + y^2)^3 = (4x^2 + 2y^2)^2$$

Find the zeros of the polar equation $r = 1 + 2 \sin 2\theta$ for $\theta \in [0, 2\pi)$.

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NOTE: You must solve this problem algebraically, NOT BY TRIAL & ERROR.

$$0 = 1 + 2 \sin 2\theta$$

$$\sin 2\theta = -\frac{1}{2}$$

$$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

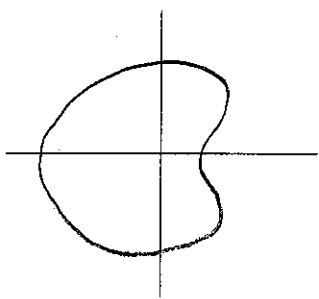
$$0 \leq \theta < 2\pi$$

$$0 \leq 2\theta < 4\pi$$

Sketch the general shape, position and direction of the polar curve $r = 7 - 4 \cos \theta$ on the axes on the right.

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NOTE: You do NOT need to find specific points. However, if the curve passes through the pole, it must be shown on the graph.



$$1 < \left| \frac{7}{-4} \right| < 2 \rightarrow \text{LIMACON WITH DIMPLE}$$

SYMMETRIC OVER X-AXIS

LARGER SIDE ON THE LEFT

Name the shapes of the following graphs.

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[a] the graph with equation $2 + 2x + 3x^2 + 18y = 0$

PARABOLA

[b] the graph with polar equation $r = -7 - 7 \cos \theta$

CARDIOID $|\frac{-7}{-7}| = 1$

[c] the graph with polar equation $r = \frac{15}{7 - 5 \cos \theta}$

ELLIPSE $e = \frac{5}{7}$

[d] the graph with polar equation $\theta = 2$

LINE

[e] the graph with equation $8 + 4x + 2x^2 + 18y - 3y^2 = 0$

HYPERBOLA

[f] the graph with polar equation $r = -5 + 2 \cos \theta$

CONVEX LIMACON

[g] the locus of points in the plane that are six times as far from $(4, 1)$ as they are from $x = 7$

P

F

D

HYPERBOLA

$PF = 6 * PQ \rightarrow \frac{PF}{PQ} = 6 = e$

Consider the conic with rectangular equation $9x^2 + 4y^2 + 72x - 8y + 4 = 0$.

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[a] Find the co-ordinates of the vertex/vertices.

$$9x^2 + 72x + 4y^2 - 8y = -4$$

$$9(x^2 + 8x) + 4(y^2 - 2y) = -4$$

$$9(x^2 + 8x + 16) + 4(y^2 - 2y + 1) = -4 + 144 + 4$$

$$9(x+4)^2 + 4(y-1)^2 = 144$$

$$\frac{(x+4)^2}{16} + \frac{(y-1)^2}{36} = 1$$

ELLIPSE 

CENTER $(-4, 1)$

VERTICES $(-4, 1 \pm 6)$

$= (-4, 7), (-4, -5)$

[b] Find the co-ordinates of the focus/foci.

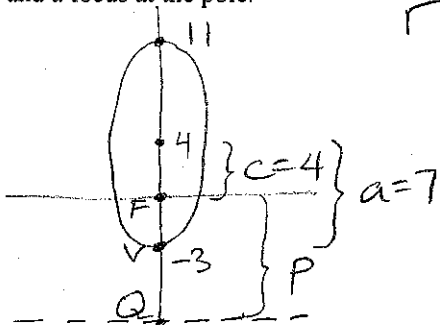
$$c^2 = 36 - 16 = 20$$

$$c = 2\sqrt{5}$$

FOCI $(-4, 1 \pm 2\sqrt{5})$

Find the **polar** equation of the ellipse with vertices with **polar** co-ordinates $(11, \frac{\pi}{2})$ and $(3, \frac{3\pi}{2})$, and a focus at the pole.

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$$r = \frac{ep}{1 - e \sin \theta}$$

OR

$$11 = \frac{ep}{1 - e \sin \frac{\pi}{2}} \rightarrow 11 = \frac{ep}{1 - e}$$

$$3 = \frac{ep}{1 - e \sin \frac{3\pi}{2}} \rightarrow 3 = \frac{ep}{1 + e}$$

$$ep = 11 - 11e = 3 + 3e$$

$$8 = 14e$$

$$e = \frac{4}{7}$$

$$\frac{4}{7}p = 3 + 3\left(\frac{4}{7}\right) = \frac{33}{7}$$

$$p = \frac{33}{4}$$

$$\text{CENTER} = (0, \frac{11-3}{2})$$

$$= (0, 4)$$

$$e = \frac{c}{a} = \frac{4}{7}$$

$$e = \frac{VF}{VQ} = \frac{3}{p-3} = \frac{4}{7}$$

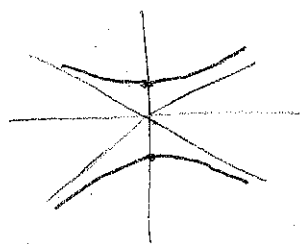
$$21 = 4p - 12$$

$$p = \frac{33}{4}$$

$$r = \frac{\frac{4}{7} \cdot \frac{33}{4}}{1 - \frac{4}{7} \sin \theta} = \frac{33}{7 - 4 \sin \theta}$$

Find the **rectangular** equation of the hyperbola with foci $(0, \pm 2)$ and asymptotes $y = \pm \frac{3}{4}x$.

SCORE: ____ / 20 PTS



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{a}{b} = \frac{3}{4} \rightarrow a = \frac{3}{4}b$$

$$a^2 + b^2 = 2^2$$

$$\left(\frac{3}{4}b\right)^2 + b^2 = 4$$

$$\frac{25}{16}b^2 = 4$$

$$b^2 = \frac{64}{25}$$

$$b = \frac{8}{5}$$

$$a = \frac{6}{5}$$

$$\frac{y^2}{\frac{36}{25}} - \frac{x^2}{\frac{64}{25}} = 1$$

OR

$$\frac{25y^2}{36} - \frac{25x^2}{64} = 1$$

Consider the polar equation $r = 3 + \cos 2\theta$.

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The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ and $(-r, \pi - \theta)$

- [a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

$$(r, -\theta): r = 3 + \cos 2(-\theta)$$

$$r = 3 + \cos(-2\theta)$$

$$r = 3 + \cos 2\theta$$

SYMMETRIC OVER POLAR AXIS

Type of symmetry	Conclusion
Over the pole	SYMMETRIC
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC

$$(r, \pi - \theta): r = 3 + \cos 2(\pi - \theta)$$

$$r = 3 + \cos(2\pi - 2\theta)$$

$$r = 3 + \cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta$$

$$r = 3 + \cos 2\theta$$

SYMMETRIC OVER $\theta = \frac{\pi}{2}$

AUTOMATICALLY SYMMETRIC OVER POLE

- [b] Draw the graph of this polar equation by plotting points for as few θ -values as needed, and using symmetry to complete the graph.
List the polar co-ordinates (in ordered pair notation) of all points you needed to plot.

ONLY NEED TO PLOT POINTS
IN 1 QUADRANT

θ	r	
0	4	(4, 0)
$\frac{\pi}{6}$	3.5	(3.5, $\frac{\pi}{6}$)
$\frac{\pi}{4}$	3	(3, $\frac{\pi}{4}$)
$\frac{\pi}{3}$	2.5	(2.5, $\frac{\pi}{3}$)
$\frac{\pi}{2}$	2	(2, $\frac{\pi}{2}$)

